

# Nucleon Resonance Effects in $pp \rightarrow pp\pi^0$ near Threshold

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## Abstract

The role of the low lying nucleon resonances beyond the  $\Delta(1232)$  in the reaction  $pp \rightarrow pp\pi^0$  near threshold is shown to be numerically significant by a calculation, which takes into account the pion re-scattering contribution described by chiral perturbation theory and the short-range mechanisms that are implied by the nucleon-nucleon interaction model. The intermediate  $N(1440)$  ( $P_{11}$ ) resonance is excited by the short-range exchange mechanisms, while the  $N(1535)$  ( $S_{11}$ ) and  $N(1520)$  ( $D_{13}$ ) resonances are excited by  $\eta$  and  $\rho$  meson exchange, respectively. The  $P_{11}$  increases the calculated cross section, whereas the  $S_{11}$  and  $D_{13}$  resonances decrease it. The calculation takes full account of the initial and final state interactions.

## I. INTRODUCTION

The cross section for the reaction  $pp \rightarrow pp\pi^0$  near threshold is exceptionally sensitive to short-range exchange mechanisms in the two-nucleon system, because the main pion exchange term is ruled out by isospin conservation in the two-nucleon system [1–3]. Pion production on a single nucleon underpredicts the empirical cross section [4–6] by a large factor [7]. Short-range exchange mechanisms that are implied by the nucleon-nucleon interaction enhance the cross section [1]. The role of the residual pion exchange mechanisms, which are not necessarily small when one pion is virtual, remains contentious as conventional phenomenological meson field theory models and chiral perturbation theory (ChPT) disagree on the sign of the pion exchange amplitude [2,3,8,9].

Systematic amplitude analysis of the reactions  $pn \rightarrow pp\pi^-$  and  $pp \rightarrow pn\pi^+$  indicates that the short-range mechanisms, which enhance the (small) cross section obtained from the single nucleon pion production mechanisms, have to dominate over the pion exchange amplitude given by chiral perturbation theory, because empirically the phase of the amplitudes for production of  $S$  and  $P$  wave pions has to be the same [10]. In this situation it appears natural to investigate the role of such other short-range mechanisms, which should be expected to contribute to this reaction, and which may be calculated with some degree of confidence.

The most obvious additional short-range mechanisms are those which involve transition couplings between different exchanged mesons, and those that involve excitation of intermediate virtual nucleon resonances by short-range exchange mechanisms. The most obvious of the former class of effects were estimated – and found to be non-negligible – in Ref. [11]. The role of intermediate  $\Delta(1232)$  resonances has been investigated in Refs. [3,12], and found to be significant despite the threshold suppression factor. The role of the intermediate  $N(1440)$  ( $P_{11}$ ) resonance excited by scalar and vector exchange mechanisms was considered in Ref. [11] and found to be small, although the small magnitude of the result depended on a rather uncertain estimate for the coupling of the effective scalar field to the  $N(1440)$  [13].

We here consider in addition the role of the intermediate  $N(1535)$  ( $S_{11}$ ) and  $N(1520)$  ( $D_{13}$ ) resonances, which form the lowest “ $P$ –shell” multiplet in the baryon spectrum. The former is excited both by pion and – in particular – by  $\eta$ –meson exchange. The latter is excited by pion as well as  $\rho$ –meson exchange. The pion exchange contributions are presumably included in the “off-shell” ChPT  $\pi N$  amplitude, and therefore do not require explicit evaluation. The  $\eta$ – and  $\rho$ –meson exchange contributions have to be derived separately. Systematic calculation of these contributions calls for employment of a boson exchange model for the nucleon-nucleon interaction so as to avoid additional model dependence through coupling constants, meson propagators, and form factors. For this reason we carry out the calculation by using the “Bonn-B” potential model [14].

In order to obtain an illustrative as well as quantitatively realistic description of the resonance contributions to the near threshold cross section for the reaction  $pp \rightarrow pp\pi^0$ , we perform the cross section calculation with full account of the nucleon-nucleon interaction in the initial and final states.

We performed the calculations of all contributions with the exact kinematics, i.e., we did not use the popular “frozen kinematics approximation” in which threshold kinematics is used also at energies above threshold. Calculations using this approximation, as well as with another approximation concerning the energy dependence of the two-nucleon T-matrix, are, however, presented for comparison.

In addition, we take explicitly into account the kinematically determined energy of the exchanged pion in the ChPT amplitude for the pion exchange contribution to the pion production amplitude, a correction term that was found to be significant in Ref. [15]. With conventional estimates for the parameters that describe these exchange mechanisms, we obtain a fairly satisfactory description of the cross section for  $pp \rightarrow pp\pi^0$  in the near-threshold region up to about 300 MeV laboratory kinetic energy without nucleon resonance and explicit short-range effects associated with meson transition couplings. This differs from other similar calculations based on other nucleon-nucleon interaction models, and reflects the extreme sensitivity to the details of the short-range parts of the interaction models.

The contributions from the  $P_{11}$  and  $S_{11}$  resonances are small, and without further short-range contributions would lead to somewhat too large cross section values above 300 MeV. The smallness of the contribution from the  $P_{11}$  resonance agrees with the finding in Ref. [11]. The smallness of the contribution from the  $S_{11}$  is mainly due to the smallness of the  $\eta$ -nucleon coupling constant.

Choosing positive signs for the coupling parameters for all resonances except the  $D_{13}$ , that particular resonance contributes with the opposite sign in comparison with the other two resonances, and taking it into account actually improves the calculated result. To this net result one may add the contributions from the intermediate  $\Delta$  resonance and from the short-range mechanisms associated with mesonic transition couplings, which however, when combined amount to a very small correction, because of their tendency for cancellation.

This article is divided into 5 sections. In section 2, a brief review of the least contentious pion production mechanisms that have been considered in the literature is given, including a simplified derivation of the short-range mechanisms associated with the nucleon-nucleon interaction. The resonance excitation amplitudes are derived in section 3. The numerical results are reported in section 4, while section 5 contains a concluding discussion.

## II. PION PRODUCTION AND THE NUCLEAR AXIAL CURRENT

## A. Chiral Symmetry Constraint

The absence of parity doubling of the experimental hadron spectra at low energies implies that the approximate chiral symmetry of  $QCD$  is realized in the hidden mode, and that the low-mass pseudoscalar meson octet  $\pi, K, \eta$  has to represent the associated Goldstone bosons. It follows that the coupling of this meson octet to hadrons has to vanish with the four-momenta of these mesons. The coupling of pions to a nuclear system therefore has to have the general form

$$\mathcal{L} = -\frac{1}{f_\pi} \partial_\mu \vec{\pi} \cdot \vec{A}_\mu, \quad (1)$$

where  $\vec{A}_\mu$  is the axial (flavor octet) current density of the nuclear system and  $f_\pi$  is the pion decay constant [16,17].

The interaction (1) implies that near-threshold pion production off nuclei is governed by the axial charge density operator, which is known to be dominated by two-nucleon mechanisms [18–20]. In this regard, the reaction  $pp \rightarrow pp\pi^0$  near threshold forms a special case, because as the main (isospin antisymmetric) pion exchange contribution is eliminated, the reaction is governed by short-range mechanisms.

These short-range mechanisms fall into three categories. The first is associated with the isospin symmetric pion exchange amplitude, which, while of vanishingly small significance for elastic pion scattering, may be large for the off-shell amplitude involved in pion production off nuclear systems. The second category are the short-range contributions to the axial charge operator that are implied by the nucleon-nucleon interaction model, which has to be employed to calculate the nuclear wave functions. They are represented in Fig. 1. The final category are the additional model-dependent short-range contributions that are associated with excitation of virtual intermediate nucleon resonances by short-range mechanisms (Fig. 2), and short-range mechanisms that are associated with mesonic transition couplings.

## B. Pion exchange contribution

The isospin symmetric pion exchange contribution to the pion production amplitude was derived by means of ChPT in Refs. [2,3], and found to yield an amplitude that disagrees in sign with that obtained by phenomenological meson field theory models [8,9]. We shall here employ the ChPT amplitude given in Ref. [3], but treat the pion energy determined kinematically exactly as in Ref. [15]. The higher-order loop contributions that involve two nucleons considered in Ref. [21], will be assumed to form part of the short-range contributions that are implied by the nucleon-nucleon interaction model described below.

The one-nucleon and the isospin symmetric pion exchange contribution to the axial charge density operators then have the expressions [1]

$$A_0^a(1) = -f_\pi \frac{f_{\pi NN}}{m_\pi} \vec{\sigma} \cdot \vec{v} \tau^a, \quad (2)$$

$$A_0^a(\pi) = f_\pi \frac{f_{\pi NN}}{m_\pi^2} \frac{8\pi\lambda_1}{\omega_\pi} \frac{\vec{\sigma}^2 \cdot \vec{k}_2}{m_\pi^2 + \vec{k}_2^2 - \omega_k^2} f(\vec{k}^2) \tau_2^a + (1 \leftrightarrow 2). \quad (3)$$

Here,  $(\vec{k}_2, \omega_k)$  is the 4-momentum of the exchanged pion that is absorbed by nucleon 2 ( $\vec{k}_2 = \vec{p}_2' - \vec{p}_2$ ), and  $f$  is a phenomenological form factor that dampens out high values of the exchanged momentum. We use the parametrization of the Bonn potential [14],

$$f(\vec{k}^2) = \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + \vec{k}^2} \right)^2, \quad (4)$$

with the pion cut-off mass  $\Lambda_\pi = 1.3$  GeV. Note that by the Goldberger-Treiman relation  $g_A/2f_\pi = f_{\pi NN}/m_\pi$ , where  $f_{\pi NN}$  is the  $\pi NN$  pseudovector coupling constant ( $f_{\pi NN} \simeq 1$ ).

The coefficient  $\lambda_1$  is determined by the  $S$ -wave  $\pi N$  scattering lengths  $a_1, a_3$  for physical  $\pi N$  scattering near threshold as

$$\lambda_1 = -\frac{1}{6} m_\pi \left( 1 + \frac{m_\pi}{m_N} \right) (a_1 + 2a_3). \quad (5)$$

Here,  $m_N$  is the nucleon mass. Extant empirical values for the scattering lengths are consistent with  $\lambda_1 = 0$ . For the off-shell  $\pi N$  scattering amplitude, appropriate to pion production associated with pion exchange, ChPT gives the following expression for  $\lambda_1$  [2,3]:

$$\lambda_1 = \frac{m_\pi^3}{4\pi f_\pi^2} \left[ 2c_1 + \left( c_2 + c_3 - \frac{g_A^2}{8m_N} \right) \frac{\omega_q \omega_k}{m_\pi^2} - c_3 \frac{\vec{q} \cdot \vec{k}_2}{m_\pi^2} \right], \quad (6)$$

where  $\vec{q}$  is the momentum of the produced pion and  $\omega_k = E_2' - E_2$ . The coefficients  $c_j$  are determined by the empirical or empirically extracted values for the two  $S$ -wave scattering lengths, the  $\sigma$  term, and the axial polarizability for  $\pi N$  scattering. We here use the values  $c_1 = -0.87$  GeV<sup>-1</sup>,  $c_2 = 3.34$  GeV<sup>-1</sup>, and  $c_3 = -5.25$  GeV<sup>-1</sup> given in Ref. [22].

The energy of the exchanged pion  $\omega_k$  is determined kinematically as

$$\omega_k = \frac{\vec{k}_2 \cdot (\vec{p}_2' + \vec{p}_2)}{2m_N}, \quad (7)$$

where  $\vec{p}_2$  and  $\vec{p}_2'$  are the initial and final momenta of nucleon 2 (the one which does not emit the real pion), although it is a common approximation to set it to half of the energy of the produced pion. The limits of the validity of that approximation were investigated in Ref. [15].

### C. Short-range exchange contributions

The contributions to the axial exchange charge operator associated with the short-range components of the nucleon-nucleon interaction have been derived in Refs. [1,19]. These contributions correspond to the nonrelativistic limit of the nonsingular part of the axial current 5-point function with external leg couplings, and are colloquially referred to as nucleon-antinucleon “pair currents” or “Z-graphs”, and are represented as such by the diagrams in Fig. 1. The form of these is implied by the Poincaré invariance of the 5-point functions [23].

The numerically most significant short-range mechanisms are those that are associated with the scalar and vector exchange terms in the nucleon-nucleon interaction. The former may be derived directly, without reference to the 5-point function, in the following way.

Consider the isospin independent scalar exchange component of the nucleon-nucleon interaction, which contains the Fermi invariant “ $S$ ”. To second order in  $v/c$ , this interaction takes the form

$$v_S^+(r)S = v_S^+(r) \left( 1 - \frac{\vec{p}^2}{m_N^2} \right) - \frac{1}{2m_N^2} \frac{\partial v_S^+(r)}{r \partial r} \vec{S} \cdot \vec{L}, \quad (8)$$

where  $v_S^+(r)$  is a scalar function. The  $\vec{p}^2/m^2$  term in the spin-independent part of this interaction may be combined with the kinetic energy term in the nuclear Hamiltonian, by replacing the nucleon mass by the effective “mass operator”

$$m^*(r) = m_N \left[ 1 + \frac{v_S^+(r)}{m_N} \right]. \quad (9)$$

To first order in  $v_S^+(r)$ , the scalar component of the nucleon-nucleon interaction therefore implies the following two-body “correction” to the single nucleon axial charge operator in Eq. (2):

$$A_0^a(S) = -\frac{v_S^+(r)}{m_N} A_0^a(1) \pm (1 \rightarrow 2). \quad (10)$$

This interaction current operator coincides in form with the “scalar exchange” pair current operator derived in Refs. [1,19]. The corresponding momentum space expression is

$$A_0^a(S) = -f_\pi \frac{f_{\pi NN}}{m_\pi} \frac{v_S^+(k_2)}{m_N} \vec{\sigma}^1 \cdot \vec{v}_1 \tau_a^1 + (1 \leftrightarrow 2), \quad (11)$$

where  $v_S^+(k)$  is the Fourier transform of the scalar potential  $v_S^+(r)$  and  $\vec{v}_1 = (\vec{p}_1 + \vec{p}_1')/2m_N$ . It is completely determined by the nucleon-nucleon interaction model. Note that because the scalar exchange interaction is attractive in realistic nucleon-nucleon potentials, this exchange current contribution

implies an enhancement of the cross section over the value given by the single nucleon pion production mechanism.

The expression for the vector exchange contribution to the axial charge operator as derived from the 5-point function has been given in Refs. [1,19]:

$$A_0^a(V) = f_\pi \frac{f_{\pi NN}}{m_\pi} \frac{v_V^+(k_2)}{m_N} \left( \vec{\sigma}^1 \cdot \vec{v}_2 + \frac{i}{2m_N} \vec{\sigma}^1 \times \vec{\sigma}^2 \cdot \vec{k}_2 \right). \quad (12)$$

Here  $v_V^+(k)$  is the isospin independent vector component of the nucleon-nucleon interaction.

The axial charge operator is unique among the nuclear current operators in that its short-range vector and scalar exchange contributions add coherently rather than cancel.

A large - and the longest range - fraction of the effective scalar and vector exchange components of the nucleon-nucleon interaction is due to two-pion exchange. A direct calculation of this uncorrelated part of the two-pion exchange component has recently been attempted by means of ChPT in ref. [24]. The results of that calculation indicate that the two-pion exchange mechanisms are large, although the largest contribution arises from a hybrid  $\pi - 2\pi$  exchange diagram.

### III. NUCLEON RESONANCE CONTRIBUTIONS

#### A. The $N(1440)$ contribution

The nucleon resonance contributions to pion production in nucleon-nucleon collisions are illustrated by the Feynman diagrams in Fig. 2. The  $N(1440)$  is the lowest vibrational state of the nucleon, and as such should be excited by the same exchange mechanisms that appear in the nucleon-nucleon interaction. In particular it is expected to be excited by the effective scalar and vector fields in the nucleon [25].

The contribution to the amplitude for the reaction  $pp \rightarrow pp\pi^0$  from virtual intermediate  $N(1440)$  resonances excited by effective isospin independent scalar exchange mechanisms may be written in terms of the corresponding amplitude for the scalar exchange contribution implied by the nucleon-nucleon interaction (11) as

$$A_0^a(S, N(1440)) = K_\sigma A_0^a(S), \quad (13)$$

where the coefficient  $K_\sigma$  is defined as

$$K_\sigma = \frac{g_{\sigma NN^*}^{1440}}{g_{\sigma NN}} \frac{f_{\pi NN^*}^{1440}}{f_{\pi NN}} \frac{2m_N}{m_{N^*}^{1440} - m_N}, \quad (14)$$

in the sharp resonance approximation. Here the  $f_{\pi NN^*}^{1440}$  denotes the pseudovector coupling constant for the  $\pi NN(1440)$  vertex, and  $g_{\sigma NN}$  and  $g_{\sigma NN^*}^{1440}$

the coupling strengths for an effective scalar meson, the exchange of which represents the effective scalar field (mainly correlated two-pion) exchange interaction between the nucleons.

The pseudovector  $\pi NN(1440)$  coupling constant is determined by the experimental partial decay width for  $N(1440) \rightarrow \pi N$  for all accessible charge channels as

$$\frac{(f_{\pi NN^*}^{1440})^2}{4\pi} = \frac{1}{3} \frac{m_\pi^2 m_{N^*}^{1440}}{p(m_{N^*}^{1440} + m_N)^2 (E_N - m_N)} \Gamma [N(1440) \rightarrow \pi N] , \quad (15)$$

which results in  $(f_{\pi NN^*}^{1440})^2/4\pi \simeq 0.01$ . This value is close to that in [26], and smaller by a factor 3 than that used in ref. [25], the difference arising from the inclusion of all different charge states in the decay width calculation. In (15),  $p$  and  $E_N$  are the momentum and energy of the final nucleon, respectively.

The numerical value for the coupling constant  $g_{\sigma NN}$  is determined by the nucleon-nucleon interaction model. The value of the  $N(1440)$  coupling to the scalar field is very uncertain [13]. It was determined from the empirical fractional decay for  $N(1440) \rightarrow N(\pi\pi)_{S_{wave}}^{I=0}$  in [25] as

$$\frac{(g_{\sigma NN^*}^{1440})^2}{4\pi} = \frac{m_{N^*}^{1440}}{p(E_N + m_N)} \Gamma [N(1440) \rightarrow N(\pi\pi)_{S_{wave}}^{I=0}] , \quad (16)$$

yielding a numerical value of  $(g_{\sigma NN^*}^{1440})^2/4\pi \simeq 0.1$  for  $\Gamma[N(1440) \rightarrow N(\pi\pi)_{S_{wave}}^{I=0}] = 35$  MeV. This value will be used here. In principle, measurement of photoproduction of vector mesons and the  $N(1440)$  resonance off protons should provide fairly direct information on the magnitude of this constant. With the value above we obtain for the ratio  $g_{\sigma NN^*}^{1440}/g_{\sigma NN}$  the value 0.11, as  $g_{\sigma NN}^2/4\pi = 8.28$  in the Bonn B potential model [14].

The intermediate  $N(1440)$  resonance may also be excited by  $\omega$ -meson exchange. This contribution separates into one that arises from the charge coupling of the  $\omega$ -meson and one that arises from the current couplings [11]. This term may be expressed as an axial charge operator of the form (in momentum space)

$$\begin{aligned} A_0^a(V) = & f_\pi \frac{f_{\pi NN}^*}{m_\pi} \left( \frac{g_{\omega NN^*}^{1440}}{g_{\omega NN}} \right) \frac{4m_N}{(m_{N^*}^{1440})^2 - m_N^2} v_V^+(k_2) \\ & \times \left[ \vec{\sigma}^1 \cdot \vec{v}_1 + \frac{m_{N^*}^{1440} - m_N}{2m_N} \vec{\sigma}^1 \cdot \left( \vec{v}_2 + \frac{i}{2m_N} \vec{\sigma}^2 \times \vec{k}_2 \right) \right] \tau_a^1 + (1 \leftrightarrow 2) . \end{aligned} \quad (17)$$

Here,  $v_V^+(k)$  is the coefficient of the isospin independent vector exchange component of the nucleon-nucleon interaction, and  $g_{\omega NN^*}^{1440}$  and  $g_{\omega NN}$  are the  $\omega NN(1440)$  and  $\omega$ -nucleon coupling constants, respectively. The momentum transfer to nucleon 2 is denoted  $\vec{k}_2$ . Following Ref. [25], we assume that the ratio between the scalar and vector coupling constants are the same for the  $N(1440)N$  couplings as for the diagonal nucleon couplings. Thus we take  $g_{\omega NN^*}^{1440}/g_{\sigma NN^*}^{1440} = g_{\omega NN}/g_{\sigma NN} = 1.55$ .



It is obvious from the expression (17) that the first term on the rhs, which is that arising from charge coupling, goes against that of the scalar exchange contribution (11). This suggests that they should be considered together. The resulting partial cancellation makes the net contribution of the intermediate  $N(1440)$  resonance small.

## B. The $N(1535)$ contribution

The  $N(1535)$  resonance stands out by its exceptionally large  $N\eta$  decay width, given its closeness to the threshold for  $\eta$ -decay. As a consequence, the  $\eta NN(1535)$  coupling constant has to be very large, and therefore the  $N(1535)$  excitation by  $\eta$ -meson exchange contributes a nonnegligible amount to the cross section for  $pp \rightarrow pp\pi^0$ .

To describe this contribution, we describe the  $\eta NN$  and  $\eta NN(1535)$  couplings by the Lagrangians

$$\mathcal{L}_{\eta NN} = i \frac{f_{\eta NN}}{m_\eta} \bar{\psi} \gamma_5 \gamma_\mu \partial_\mu \eta \psi, \quad (18a)$$

$$\mathcal{L}_{\eta NN(1535)} = i \frac{f_{\eta NN^*}^{1535}}{m_\eta} \bar{\psi}(1535) \gamma_\mu \partial_\mu \eta \psi + h.c. \quad (18b)$$

The  $\eta NN$  pseudovector coupling constant  $f_{\eta NN}$  is contained in the potential model, and is related to the corresponding pseudoscalar coupling constant as  $f_{\eta NN} = m_\eta g_{\eta NN} / 2m_N$ .

The coupling constant  $f_{\eta NN^*}^{1535}$  may be calculated from the partial width for  $N\eta$  decay of the  $N(1535)$  as [27]:

$$\frac{(f_{\eta NN^*}^{1535})^2}{4\pi} = \frac{m_\eta^2 m_{N^*}^{1535}}{p\omega_\eta^2(E_N + m_N)} \Gamma[N(1535) \rightarrow N\eta]. \quad (19)$$

The empirical partial decay width 67 MeV then yields the value  $(f_{\eta NN^*}^{1535})^2 / 4\pi = 0.24$ .

The  $\pi NN(1535)$  coupling has the same form as (18a), with the modification that  $\eta$  is replaced by  $\vec{\tau} \cdot \vec{\phi}$ ,  $m_\eta$  by  $m_\pi$  and  $\omega_\eta$  by  $\omega_\pi$ . The  $\pi NN(1535)$  coupling constant may then be calculated from the partial width for  $N\pi$  decay of the  $N(1535)$  in analogy with (19). From the empirical decay width 67.5 MeV we then obtain the value  $(f_{\pi NN^*}^{1535})^2 / 4\pi = 0.0021$ . The smallness of this value has been explained by chiral symmetry arguments [28].

The  $N(1535)$  resonance excited by  $\eta$  exchange gives rise to a contribution to the amplitude for  $S$ -wave pion production in the reaction  $pp \rightarrow pp\pi^0$ , which - to lowest order in the sharp resonance approximation, and with neglect of delta function terms - may be expressed as an axial exchange charge operator:

$$A_0^a(\eta) = -f_\pi \frac{\omega_\pi f_{\pi NN^*}^{1535}}{m_\pi m_\eta^2} \frac{f_{\eta NN^*}^{1535}}{f_{\eta NN}} \frac{\vec{\sigma}^2 \cdot \vec{k}_2 \vec{\tau}_a^1}{m_{N^*}^{1535} - m_N} \left( \frac{m_\eta}{2m_N} \right)^2 v_{PS}^+(k_2) + (1 \leftrightarrow 2). \quad (20)$$

Here, the function  $v_{PS}^+$  is the isospin independent pseudoscalar exchange component of the nucleon-nucleon interaction, which in a one-boson exchange interaction model has the form

$$v_{PS}^+(k) = \frac{g_{\eta NN}^2}{m_\eta^2 + \vec{k}^2}, \quad (21)$$

with  $g_\eta = 2m_N f_{\eta NN}/m_\eta$ .

### C. The $N(1520)$ contribution

The  $N(1520)$  is the spin  $3/2^-$  partner of the  $N(1535)$ . Although almost degenerate in mass, its structure - as indicated by its decay pattern - is quite different. The outstanding feature is the substantial decay branch (10-15%) to the  $N\rho$  channel. As this decay kinematically can reach only the lower end of the  $\rho$  meson spectrum, the corresponding coupling constant has to be very large. Therefore, the  $N(1520)$  resonance excited by  $\rho$ -meson exchange should be expected to give a non-negligible contribution to the reaction  $pp \rightarrow pp\pi^0$ .

The coupling of pions and  $\rho$ -mesons to the  $N(1520)$  resonance may be described by the Lagrangians

$$\mathcal{L}_{\pi NN(1520)} = i \frac{f_{\pi NN^*}^{1520}}{m_\pi} \bar{\psi}_\mu(1520) \gamma_5 \vec{\tau} \cdot \partial_\mu \vec{\phi} \psi + h.c., \quad (22a)$$

$$\mathcal{L}_{\rho NN(1520)} = i g_{\rho NN^*}^{1520} \bar{\psi}_\mu(1520) \left( \delta_{\lambda\mu} + \frac{\partial_\mu}{m_{N^*}^{1520} - m_N} \gamma_\lambda \right) \vec{\rho}_\lambda \cdot \vec{\tau} \psi + h.c. \quad (22b)$$

The second term in (22b) is required by the transversality of the  $\rho$ -field.

The  $\pi NN(1520)$  coupling constant may be determined from the partial width for the decay  $N(1520) \rightarrow N\pi$  to all charge channels as [27]:

$$\frac{(f_{\pi NN^*}^{1520})^2}{4\pi} = \frac{m_{N^*}^{1520} m_\pi^2}{p^3 (E_N - m_N)} \Gamma [N(1520) \rightarrow N\pi]. \quad (23)$$

This yields  $(f_{\pi NN^*}^{1520})^2/4\pi = 0.19$  and  $f_{\pi NN^*}^{1520} = 1.6$ .

The  $\rho NN(1520)$  coupling constant may be estimated from the partial width for  $\rho N$  decay of the  $N(1520)$ , which is about 24 MeV [29]. To lowest order in  $p/m_N$ , this decay rate may be written as

$$\Gamma [N(1520) \rightarrow N\rho] = \frac{2}{9} \frac{(g_{\rho NN^*}^{1520})^2}{4\pi} \frac{p}{m_{N^*}^{1520}} \frac{m_N}{E_N} (E_N + m_N). \quad (24)$$

This expression, however, only applies for kinematically allowed decays. In the case of  $\rho$  decay of the  $N(1520)$  only the lower tail of the  $\rho$ -meson spectrum between  $2m_\pi$  and the kinematical threshold at 581 MeV is accessible. If the  $\rho$ -meson spectrum is described by a Lorentzian centered at 770 MeV with a full width of 151 MeV [29], the weight of the kinematically allowed part of

the spectrum for  $\rho$ -meson decay is only 0.076. That factor should then be included on the rhs of Eq. (24).

To determine the kinematical factors  $p$  and  $E_N$  in (24), a  $\rho$ -meson mass has to be specified. This will be taken to be 480 MeV, which is the average of the kinematically allowed mass value range, weighted with the  $\rho$ -meson spectrum. Using this mass value, and taking into account the probability factor 0.076, Eq. (24) gives the values  $(g_{\rho NN^*}^{1520})^2/4\pi = 0.43$ , and  $g_{\rho NN^*}^{1520} = 2.3$ . This magnitude of this coupling constant is only about one half that (-5.3) used in Ref. [30] (The decay width does not determine the sign of this coupling constant).

The isospin symmetric part of the contribution of the intermediate  $N(1520)$  resonance excited by  $\rho$ -meson exchange to the amplitude for  $S$ -wave pion production may be expressed – to lowest order in  $1/m$  – as an axial exchange charge operator with the form

$$A_0^a(\rho) = f_\pi \frac{2}{3} \frac{f_{\pi NN^*}^{1520}}{m_\pi(m_{N^*}^{1520} - m_N)} \left( \frac{g_{\rho NN^*}^{1520}}{g_{\rho NN}} \right) v_V^-(k_2) \vec{\sigma}^1 \cdot \left[ \left( \frac{m_N}{(m_{N^*}^{1520})^2 - m_N^2} \right) \vec{k}_2 - \frac{2m_N}{m_{N^*}^{1520} + m_N} \vec{v}_1 + \vec{v}_2 + \frac{i\kappa_\rho}{2m_N} (\vec{\sigma}^2 \times \vec{k}_2) \right] \tau_a^2 + (1 \leftrightarrow 2). \quad (25)$$

Here, the function  $v_V^-(k)$  is the isospin dependent part of the vector exchange component of the nucleon-nucleon interaction, which in a boson exchange model would have the form

$$v_V^-(k) = \frac{g_{\rho NN}^2}{m_\rho^2 + \vec{k}^2}. \quad (26)$$

The parameter  $\kappa_\rho$  is the  $\rho NN$  tensor coupling constant. With the positive sign for all resonance coupling constants, this contribution would have the same sign as that of the  $N(1440)$  resonance. The data, however, favor the negative value for the  $\rho NN(1520)$  coupling (as used in Ref. [30]), as a consequence of which its contribution opposes the effect of the  $N(1440)$ .

#### IV. NUMERICAL RESULTS

The calculated values of the total cross section for the reaction  $pp \rightarrow pp\pi^0$  are shown in Fig. 3 for energies near threshold. The *cumulative* contributions of the different components in the transition amplitude are shown successively.

The short-dashed curve is that obtained with the single nucleon axial current operator (2) combined with the pion exchange term (3). This result confirms the calculations of Refs. [2,3] that there is a strong destructive interference between the impulse and rescattering terms. The pion exchange contribution was calculated with full account of the energy of the exchanged pion as in Ref. [15].

The short-dash-dotted curve in Fig. 3 includes the short-range contributions associated with the scalar (11) and vector exchange (12) contributions to

the nucleon-nucleon interaction (“Z-graphs”). Here the potential coefficients  $v_S^+$  and  $v_V^+$  were taken from the Bonn B potential [14] by the method described in Ref. [25]. When using this potential model, the cumulative cross section obtained with inclusion of these short-range effects are close to the empirical values given in Refs. [4–6]. This result also differs from those obtained earlier with other potential models, where additional short-range mechanisms had to be included in order to reach the empirical cross section [11].

Turning then to the contributions of the nucleon resonances, it is first noted that the result for the  $N(1440)$  resonance (dashed curve) is sizable, when calculated with the coupling constants used in Ref. [25]. Since the value for the  $\sigma NN(1440)$  coupling constant could still be significantly larger [13,31,32], this contribution could, accordingly, be even more pronounced, and the result reported here may be interpreted as a lower limit. The resonance  $N(1535)$  (short-dotted) has a negligible effect and the corresponding curve almost coincides with the dashed curve from the  $N(1440)$  resonance. The  $N(1520)$  (if the  $\rho NN(1520)$  coupling constant is negative), as can be seen from the dashed and the solid curves, counteracts the effect of the Roper resonance  $N(1440)$ , and therefore the net effect of these resonances is small.

When all resonances are included, the calculated cross section exceeds the empirical values at the higher end of the energy range considered. This feature suggests that there should be further short range contributions (c.f. [11]) that may play a role. In ref. [1], for instance, the small contributions from the isospin  $I=1$  scalar and vector exchanges were considered. In addition, the relativistic corrections to the rescattering amplitudes will become more significant with increasing pion momentum.

Since the signs of the mesonic resonance transition couplings are not fixed by the empirical partial widths and decay rates, we show in Fig. 4 the uncertainty implied by the uncertainty in sign and magnitude of the  $\rho NN(1520)$  coupling in the calculated cross section. The figure indicates that with positive signs for the couplings of the other resonances, the negative sign for the  $\rho NN(1520)$  vertex coupling is favored.

In order to illustrate the sensitivity of the calculated cross section to the details of the dynamics and the conventional approximations, we also show in Figs. 5 and 6 how the results for the impulse and the pion exchange terms change if one uses the so-called “frozen kinematics approximation” applied to those operators. In this approximation, the production operators are evaluated in exact threshold kinematics, even for energies above threshold. The calculation of the cross-section is simplified in this case, since the pion production amplitude does not depend on the pion momentum variable of integration.

Clearly, using frozen kinematics is not a good approximation to the exact calculation in either case, neither for the impulse term of Fig. 5, nor for the pion re-scattering term of Fig. 6, in agreement with ref. [15]. The discrepancy – not surprisingly – worsens the further one goes from threshold.

We also consider the approximation that the energy dependence of the

half-off-shell T-matrix, in the final state, is replaced by the energy dependence of the on-shell T-matrix [33]. It is based on the observation that, although the on-shell and half-off-shell T-matrix elements entering in the calculation of pion production can be very different, their variation with energy is very similar and can be isolated into a factor dominated by the two-nucleon phase shift. It suffices then to calculate the half-off-shell T-matrix at only one energy, e.g., at threshold, and the factor containing the phase shift takes care of the extension to all other energies. This simplifies the phase-space integration, without sacrificing the final state interaction.

Figures 5 and 6 display the effect of this approximation in the following way: the frozen kinematics approximation has been calculated on one hand using the exact half-off-shell two-nucleon T-matrix (dotted curve), and on the other hand with the described energy-dependence factorization (dashed curve). Therefore, the dotted curve has to be compared with the dashed curve in order to assess the quality of this approximation, *not* with the solid line representing the result without any of the considered approximations.

The energy-dependence approximation works somewhat better than the frozen kinematics approximation. As to be expected, the discrepancy increases with the distance in energy to the point at which the half-off-shell T-matrix has been calculated exactly – in this case at threshold.

The frozen kinematics calculation overestimates the impulse term, but underestimates the pion re-scattering term. It is likely that partial cancellations will occur when the two terms are calculated together, resulting in a smaller discrepancy. However, accidental cancellations of errors should not be interpreted in favor of an approximation. It is quite clear from these results that the frozen kinematics approximation should be avoided.

## V. CONCLUSIONS

The results obtained here indicate that the net effect of the orbitally excited intermediate nucleon resonances with mass below 1.6 GeV, that are excited by short-range exchange mechanisms, to the calculated cross section for the reaction  $pp \rightarrow pp\pi^0$  is small. The resonance coupling parameters were all determined phenomenologically from experimental decay widths and branching ratios. The individual resonance contributions are not small, however, and the size of the net effect depends on the sign of the transition coupling constants. The data seem to favor a negative  $\rho NN(1520)$  coupling constant.

The results also support the view that the dominant dynamical effects in the reaction  $pp \rightarrow pp\pi^0$  are (a) the single nucleon operator (2) [7], (b) the short-range exchange mechanisms incorporated in the short-range part of realistic nucleon interactions (11) and (12) [1], and (c) the pion exchange term (3). Of these it is presently the last one that is most contentious [2,3,15] and which calls for further study. Additional short-range contributions associated with transition couplings between heavy mesons are possibly also important,

and interesting because of their other phenomenological implications [11].

The short-range exchange mechanisms associated with the short-range components of the nucleon-nucleon interaction may be derived directly from the nonrelativistic limit of the nonsingular part of the axial current 5-point functions with external leg couplings, and are completely determined by the nucleon-nucleon interaction [1,19]. These short-range mechanisms were here determined from the “Bonn B” boson exchange model for the nucleon-nucleon interaction model. Their magnitude is larger than what the “power counting rules” of Chiral Perturbation Theory naively would suggest, namely that the resonance mechanisms should be larger than the re-scattering term and the “5-point contact” terms [3]. This is a consequence of the large effective momentum scale involved in the exchange mechanisms.

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## FIGURES

FIG. 1. Pair term or "Z-graph" representation of the axial exchange charge operator contributions to pion production.

FIG. 2. Nucleon resonance contributions to pion production in nucleon-nucleon collisions.

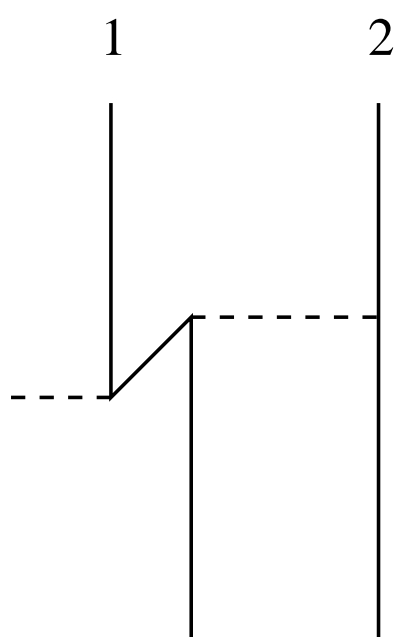
FIG. 3. Total cross section as function of the proton lab energy. The meaning of the various curves is discussed in the text.

FIG. 4. Total cross section as function of the proton lab energy. The solid curve corresponds to  $g_{\rho NN^*}^{1520} = -2.3$ , the short-dotted curve to  $g_{\rho NN^*}^{1520} = 2.3$ , the dashed curve to  $g_{\rho NN^*}^{1520} = -5.3$  and the short-dash-dotted curve to  $g_{\rho NN^*}^{1520} = -5.3$ .

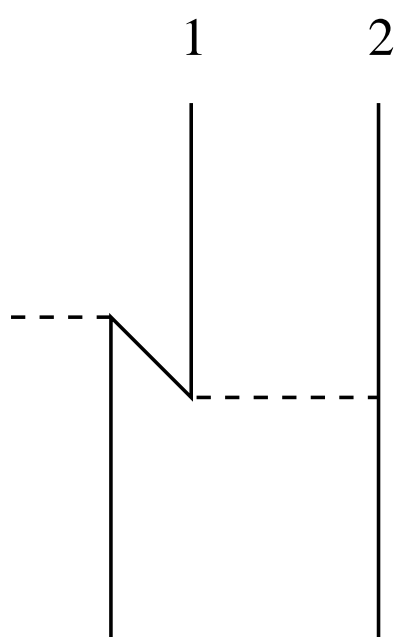
FIG. 5. Total cross section based on exact and approximate calculations of the single-nucleon term. The solid line is the exact result, the dotted line applies the "frozen kinematics approximation" ( $q=0$ ), and the dashed line refers to a calculation where, in addition, the energy-dependence of the nucleon-nucleon T-matrix is approximated, as discussed in the text.

FIG. 6. Total cross section based on exact and approximate calculations of the pion re-scattering term. The meaning of the various curves is explained in Fig.5

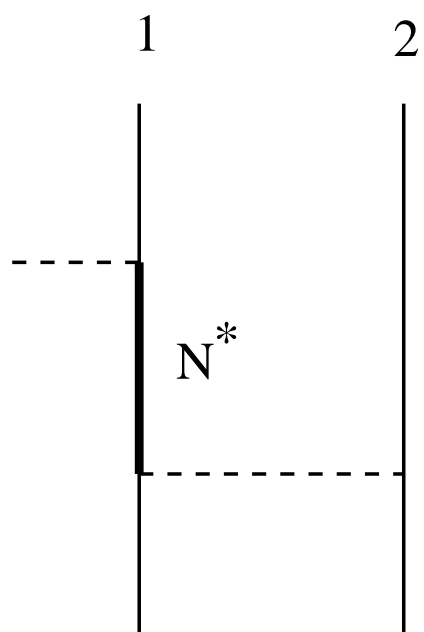




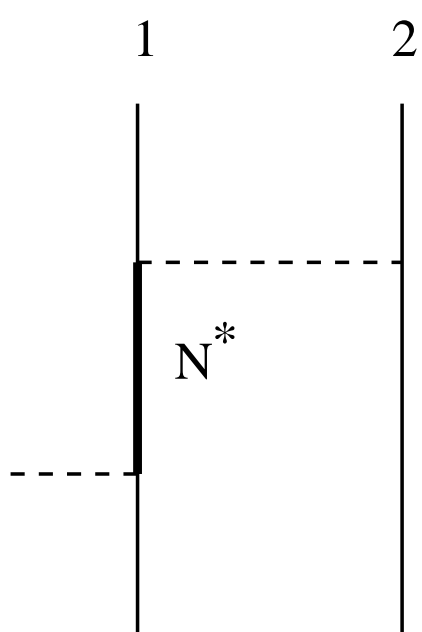
(a)



(b)



(a)



(b)

